MFM 1P - Grade Nine Applied Mathematics

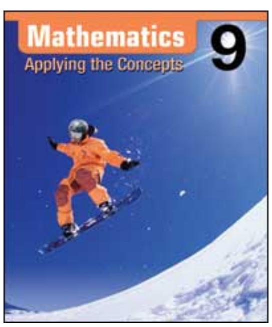
This guide has been organized in alignment with the 2005 Ontario Mathematics Curriculum. Each of the specific curriculum expectations are cross-referenced to the text book sections.

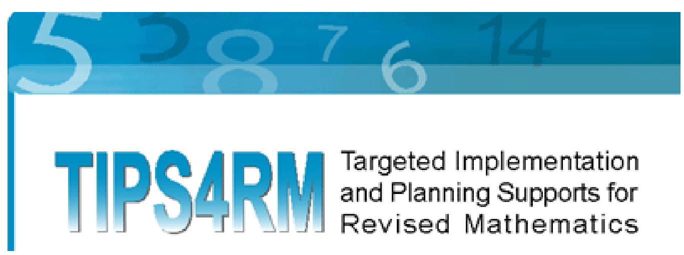
Contents

Introduction

This guide has been arranged in the order of which topics are presented in the Ontario Revised Mathematics Curriculum of 2005. Secondary schools in the Niagara Catholic District School Board as of September 2009 will use the text Pearson Math 9.

This guide was created in this fashion in the event a class uses a different primary resource, such as a textbook from another publisher, or a textbook alternative such as TIPS4RM. This may very well be the case, as the Pearson book is new to the schools this year. Many classrooms use the TIPS4RM as main resource, or may still be using the text Mathematics 9 Applying the Concepts by McGraw-Hill Ryerson.





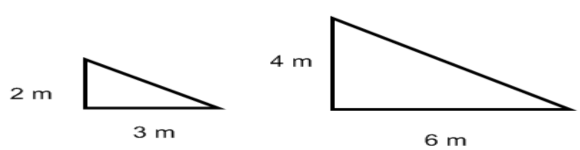
Number Sense and Algebra

Key Words:	
Algebraic Expression: a mathematical phrase made up of numbers and variables, connected by addition or subtraction signs (eg. $3x + 4$) Also called a **Polynomial**.	**Coefficient:** the number part of a term, also called the **numerical coefficient**
Constant Term: a term with only a numerical coefficient and no variable	**Distributive Property:** rule by which polynomials are multiplied, or expanded
Equation: a mathematical statement that shows two expressions are equal	**Formula:** describes an algebraic relationship that exists between two or more variables
Like Terms: terms that have identical variables	**Rate:** a certain quantity considered in relation to another quantity **e.g.** Speed is the rate at which distance changes in relation to time
Ratio: a comparison between two quantities	**Solution:** the value for the variable that make the equation true
Term: an expression formed by the product of a number and a variable.	**Unit Rate:** the quantity associated with a single unit of another quantity
Variable: a quantity whose value can change, usually represented by a letter, also called a **literal coefficient**	**Zero Pair:** two opposite integer or algebraic terms whose sum is zero

Ratios, Rates and Proportions

- **Equivalent Ratios**

2:3 = 4:6 can be visualized as equal by comparing the steepness of the two ramps



Sample Problem: A skateboard ramp is to be built with a height to base ratio of 2:3. Determine the necessary base length if the height of the ramp is to be 6 metres.

Solution: Set up and solve equivalent ratios using an unknown.

$$\frac{2}{3} = \frac{6}{x}$$

Several strategies can be shown as a means of solving for the unknown, including equivalent ratios, algebraic reasoning.

- **Comparing using unit rates**

Sample problem: Which cereal is the better buy:
A 500 g box for $1.49 or a 750 g box for $2.19 ?

Solution: Convert each to a unit rate, cost per gram, to compare the values.

- **Percent as a Ratio**

Students will have to know how to convert a fraction to a decimal. This can be used to help solve problems involving the calculation of the percent of a number. For example:

40 % of the 200 grade nine students attended the first school dance. How many is this?

40 % means 40 out of 100 or $\frac{40}{100}$ which is 0.4 in decimal form. So 0.4 × 200 = 80

*** Page 139 of Pearson Math 9 text has a one page summary of all "Need to Know" facts for this unit**

Manipulating Expressions and Solving Equations

- **Simplifying and Evaluating numerical expressions involving integers and rationals**

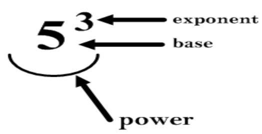
*the knowledge and skills described in this expectation are to be introduced as needed and applied and consolidated throughout the course. There is no dedicated "unit" to this material, although many teachers may take time at the beginning of the course to review these essential skills. Student's abilities in work with integers and fractions are often in need of additional support.

RECALL: B.E.D.M.A.S.

Order of Operations

Expression	Evaluation	Operation
6 + 7 x 8	= 6 + **7 x 8**	Multiplication
	= 6 + 56	Addition
	= 62	
16 ÷ 8 - 2	= **16 ÷ 8** - 2	Division
	= 2 - 2	Subtraction
	= 0	
(25 - 11) x 3	= **(25 - 11)** x 3	Brackets
	= 14 x 3	Multiplication
	= 42	

- **Substitution into and evaluation of expressions involving exponents**



Making sure students understand the difference between expressions $(-3)^4$ and -3^4

$(-3)^4 = (-3)(-3)(-3)(-3)$ **$-3^4 = -(3)(3)(3)(3)$**

With rational bases, the exponent applies to both the numerator and the denominator

$$\left(\frac{2}{3}\right)^2 = \frac{(2)^2}{(3)^2} = \frac{4}{9}$$

- **Polynomials**

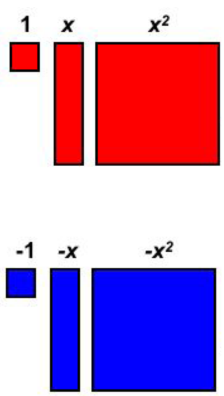
In this course a strong emphasis is made on connecting algebra to concrete materials such as algebra tiles.

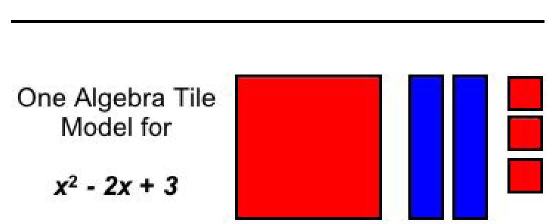
How do they help students? (from OAME website)

Algebra tiles are used to build concrete area representations of abstract algebraic concepts. The concrete representations help students become comfortable with using symbols to represent algebraic concepts. Algebra tiles are typically used to explore integers, algebraic expressions, equations, factoring, and expanding. They can also used to explore fractions and ratios. The square pieces can be used for some activities that require colour tiles.

What are they?

Algebra tiles are rectangular shapes that provide area models of variables and integers. Different pieces are used to model 1, x, x^2. Sets consist of two different colours to represent both positive and negative terms. Overhead versions are used for whole class learning opportunities. A clear plastic organizer is used to prevent tiles from moving around.





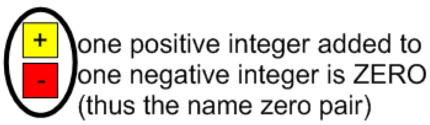
We name polynomials based on the number of terms.

		Polynomial Name
$4x$	One Term	Monomial
$3x+2$	Two Terms	Binomial
$9x^2-2x+1$	Three Terms	Trinomial

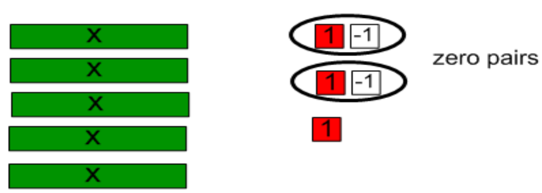
Polynomials are simplified by collecting like terms. Students are expected to work with polynomial terms that can involve up to degree three terms in one variable.

Examples: **a)** $6x - 2 + 3x + 7$ **b)** $(4x^2 + 3x + 2) + (5x^2 - 3)$

These can be illustrated to students using the algebra tiles and the concept of a ZERO PAIR.



So an example might look like: 2x + 3 + 3x – 2



leaving $5x + 1$

Expand polynomial expressions by multiplying a monomial by a polynomial, involving the same variable, using the distributive property.

Examples: **a)** $3p(-5p)$ **b)** $5c(c^2 - 6c - 1)$

- **Solve first degree equations**

Students will be introduced to different methods for solving equations

Solve by Inspection – for one step equations where the solution is evident by "just looking at it".

Examples: **a)** $a + 4 = 7$ **b)** $5y = 35$ **c)** $\frac{k}{4} = 3$

Solve using the BALANCE Method – the balance method emphasizes what you do to one side of the equation, you must also do to the other side

Illustration: **In this method, we want to have students show the steps.**

Solve:

$$2x + 3 = 11$$
$$2x + 3 - 3 = 11 - 3 \quad \text{Subtract three from both sides.}$$
$$2x = 8$$
$$\frac{2x}{2} = \frac{8}{2} \quad \text{Divide both sides by two.}$$
$$x = 4$$

A big emphasis is on the showing of steps in solving these equations. As a result, teachers may have students work as far as this balance method only. But other algebraic strategies include:

Solve using OPPOSITE Operations – this method focuses on identifying an operation, and doing the opposite to the opposite side as the path to isolation of the variable.

Solve:

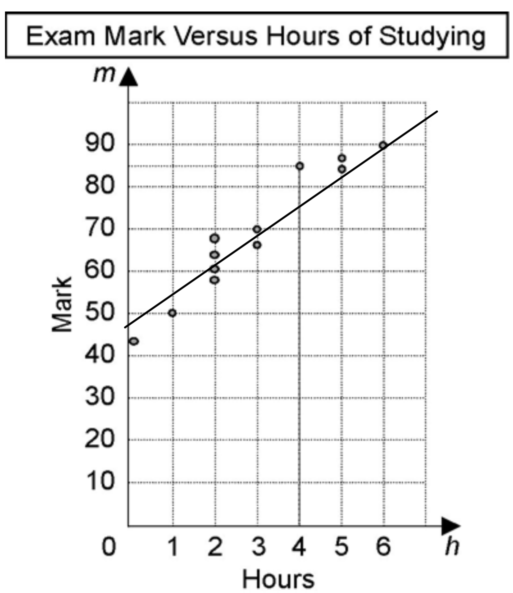
$$2x + 3 = 11$$
$$2x = 11 - 3 \quad \text{Add 3 on the left becomes Subtract 3 on the right}$$
$$2x = 8$$
$$x = \frac{8}{2} \quad \text{Multiply by 2 on left becomes Divide by 2 on right}$$
$$x = 4$$

*** Page 287 of Pearson Math 9 text has a one page summary of all "Need to Know" facts for this unit**

Linear Relations

Key Words:	
Dependent Variable: a variable that is affected by some other variable	**Direct Variation:** a relationship between two variables in which one variable is a constant multiple of the other. This multiple is called the **constant of variation.**
Extrapolate: an estimated value that lies beyond the range of measured values in a data set	**Hypothesis:** a theory or statement that is either true or false
Independent Variable: a variable that affects the value of another variable	**Interpolate:** an estimated value that lies between two measured values in a set of data
Linear Relation: a relation between two variables that forms a straight line when graphed, other wise the relation is a called a **Non-linear Relation.**	**Line of Best Fit:** the straight line that comes closest to the points on a scatter plot. The line must show the trend in the data, and have roughly an equal number of points above and below the line. Some data may be better captured by a **Curve of Best Fit.**
Partial Variation: a relationship between two variables in which the dependent variable is the sum of a constant number and a constant multiple of the independent variable	**Rate of Change:** also called the slope of the line, the ratio of rise to run between two points
Rise: the vertical difference between two points	**Run:** the horizontal difference between two points

Data Management for investigating relationships



In the **scatter plot** shown, a relationship between Exam Mark and Hours of Studying is being explored. A **hypothesis** may have been, those that study more will get better grades.

Hours is the **independent variable,** and is shown on the horizontal axis, while Mark is the **dependent variable** and is shown on the vertical axis.

A **line of best fit** is drawn to show the relational trend in the variables.

A guess of a student's mark that studied for 3.5 hours would be an example of **interpolation,** while a guess at what a student would get if they studied for 7 hours would be an **extrapolation.**

Characteristics of Linear Relations

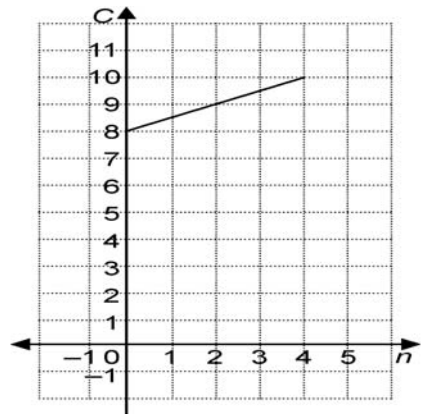
- **Constructing tables of values, graphs and equations to represent a linear relation**

A large cheese pizza costs $8.00, plus $0.50 for each topping.

a) Make a table to represent the relation.
b) Graph the relation.
c) Write an equation to represent the relation.

Number of Toppings, *n*	Cost ($), *C*
0	8.00
1	8.50
2	9.00
3	9.50
4	10.00

$$C = 0.5n + 8$$



The above is an example of a Partial Variation. The graph of a partial variation does not pass through the origin, while a Direct Variation has its graph pass through the origin.

The graph shows that indeed the relation between a cost of pizza and the number of toppings is linear, this can also be observed through an examination of first differences.

Number of Toppings, *n*	Cost ($), *C*	First Difference
0	8.00	8.5 – 8.0 = **0.5**
1	8.50	9.0 – 8.5 = **0.5**
2	9.00	9.5 – 9.0 = **0.5**
3	9.50	10.0 – 9.5 = **0.5**
4	10.00	

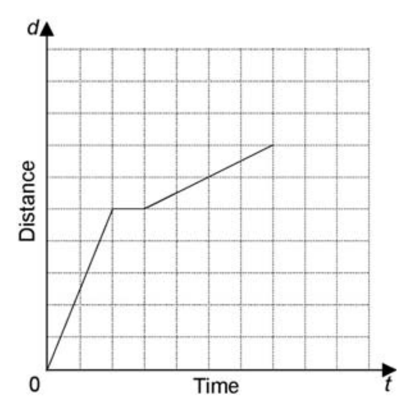
When the first differences are constant, this is property of a linear relation.

As well, if the independent variable increases by one, the first difference value will correspond to what is called the **constant of variation,** which is also the **slope** of the line.

- **Distance time graphs**

Michelle is late for school. She runs halfway to school,
then gets tired and stops for a short rest.
Then, she walks the rest of the way.
The graph to the right is a possible representation of this movement.

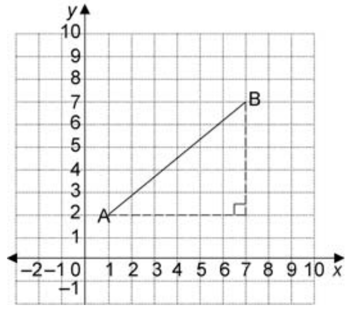
Students are to understand the meaning of direction, and rate of change of position from such a graph.



Investigating Rates of Change

- Rate of change = $\frac{\text{rise}}{\text{run}}$

  $= \frac{5}{6}$



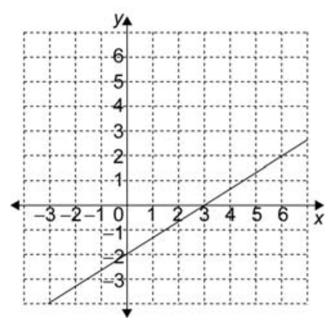
rise

run

A constant rate of change is a straight line segment on the graph.

- **Intercepts**

Identify the *x*- and *y*-intercept of the line.



x intercept is 3

y intercept is -2

The vertical or y intercept should be connected to the idea that this is the initial value in a linear relation ship. E.g. A gym membership costs $40 at sign up and then $5 per visit.
The 40 is the vertical intercept, and the 5 is the rate of change.

Thus a relation to describe this situation could be given as $C = 40 + 5v$

Using Properties of Lines to Solve Problems

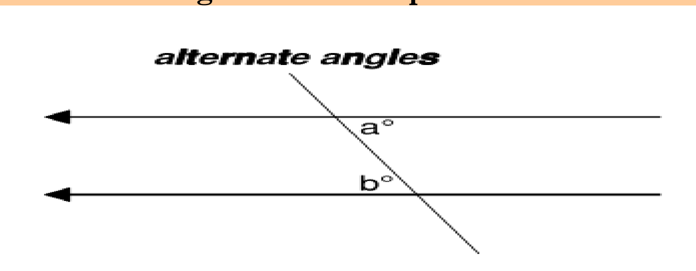
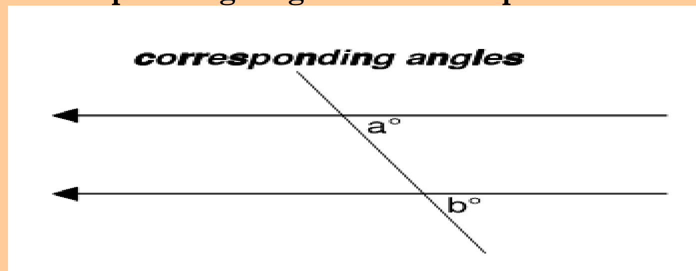
Solving a problem

A retail store offers two different hourly compensation plans:
Plan A: $9.00 per hour
Plan B: $7.50 per hour worked plus a $4.50 shift bonus.

a) Graph the linear system. When would the earnings from the two plans be the same?
b) Describe a situation under which you would choose each plan.

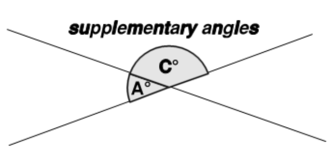
*** Pages 185 and 247 of Pearson Math 9 text have summaries of all "Need to Know" facts for this strand**

Measurement and Geometry

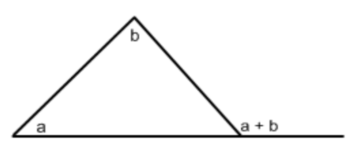
Key Words:	
Alternate Angles: form a Z pattern 	**Corresponding Angles: form a F pattern** 
Exterior Angle: angle formed on the outside of a geometric shape by extending one of the sides past a vertex	**Interior Angle:** angle formed on the inside of a polygon by two sides meeting at a vertex
Optimization: process for finding a best, or optimal value or solution. This optimal value may be a **minimum** or a **maximum**.	**Polygon:** a closed figure made up of line segments
Prism: a polyhedron with two congruent parallel bases that are connected by lateral faces that are rectangles	**Pyramid:** a polyhedron whose base is a polygon, and whose sides are triangles that meet at a common vertex
Pythagorean Theorem: in a right triangle, the square of the **hypotenuse** is equal to the sum of the squares of the other two sides (**legs**) Illustrated by the relation $c^2 = a^2 + b^2$	**Transversal:** a line intersecting two or more lines
Vertex: point where two or more sides meet	**Volume:** the amount of space that an object occupies, measured in cubic units

Investigating and Applying Geometric Relationships

- **Angle Properties**



A + C = 180 °



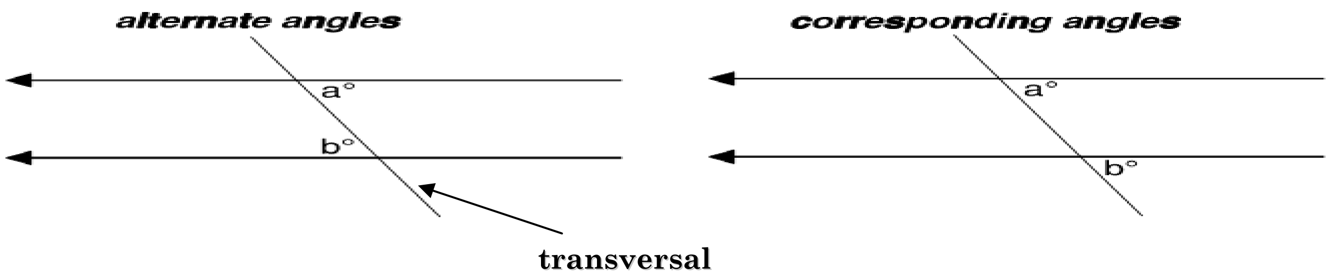
Exterior angles of a triangle

The sum of the interior angles of a polygon having n sides is equal to $(n-2) \times 180$ degrees.

The sum of the exterior angles of a polygon is 360 degrees.

A regular polygon has all interior angles equal, and the lengths of each side equal.

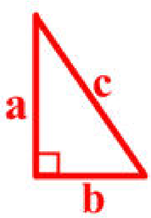
- **Parallel Line Properties**



alternate angles are equal **corresponding angles are equal**

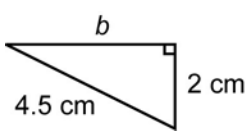
Solving Problems involving Perimeter, Area and Volume

Pythagorean Theorem



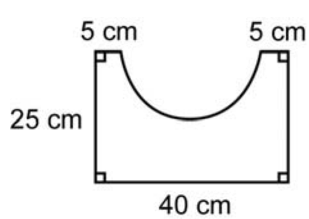
$$a^2 + b^2 = c^2$$

Students will solve problems with the above theorem.



Find the length of the unknown side.

- **Composite Figures**



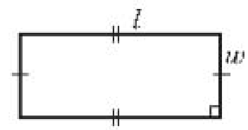
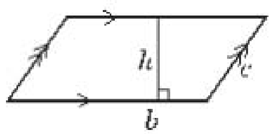
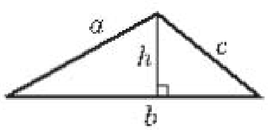
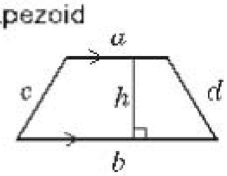
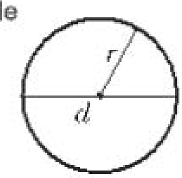
Find the perimeter and area of the above figure.

- **Two Dimensional and Three Dimensional Geometric Shapes**

The following two pages contain EQAO formula sheets containing all required formulas.

Formula Sheet

Grade 9 Applied

Geometric Figure	Perimeter	Area
Rectangle 	$P = l + l + w + w$ or $P = 2(l + w)$	$A = lw$
Parallelogram 	$P = b + b + c + c$ or $P = 2(b + c)$	$A = bh$
Triangle 	$P = a + b + c$	$A = \frac{bh}{2}$ or $A = \frac{1}{2}bh$
Trapezoid 	$P = a + b + c + d$	$A = \frac{(a + b)h}{2}$ or $A = \frac{1}{2}(a + b)h$
Circle 	$C = \pi d$ or $C = 2\pi r$	$A = \pi r^2$

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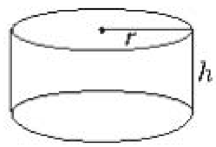
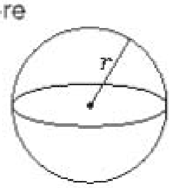
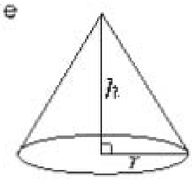
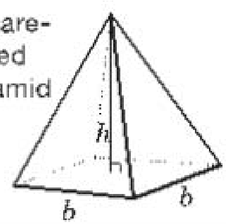
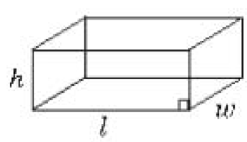
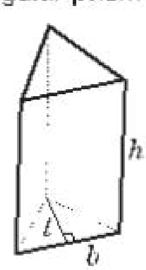
Geometric Figure	Volume
Cylinder 	$V = \text{(area of base)(height)}$ $V = \pi r^2 h$
Sphere 	$V = \frac{4}{3}\pi r^3$ or $V = \frac{4\pi r^3}{3}$
Cone 	$V = \frac{\text{(area of base)(height)}}{3}$ $V = \frac{1}{3}\pi r^2 h$ or $V = \frac{\pi r^2 h}{3}$
Square-based pyramid 	$V = \frac{\text{(area of base)(height)}}{3}$ $V = \frac{1}{3} b^2 h$ or $V = \frac{b^2 h}{3}$
Rectangular prism 	$V = \text{(area of base)(height)}$ $V = lwh$
Triangular prism 	$V = \text{(area of base)(height)}$ $V = \frac{1}{2} blh$ or $V = \frac{blh}{2}$

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Investigating Optimal Values of Measurements

- **Optimizing Rectangles**

Sample Problem: A rectangle has perimeter 20 units.

a) List all the possible whole-number dimensions of the rectangle.
b) Which dimensions produce the rectangle with maximum area?
c) Describe the shape of the rectangle with maximum area.

Sample Problem: School plans on enclosing a play ground area using 100 metres of fence. One side of the area will be bounded by the school, so the fence will be required for only three sides of the rectangle. Determine the dimensions that will maximize the area that can be enclosed.

*** Pages 37, 69 and 105 of Pearson Math 9 text have summaries of all "Need to Know" facts for this strand**