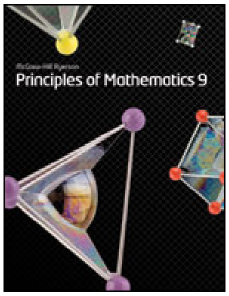
MPM 1D - Grade Nine Academic Mathematics

This guide has been organized in alignment with the 2005 Ontario Mathematics Curriculum. Each of the specific curriculum expectations are cross-referenced to the text book sections.

Contents

Introduction

This guide has been arranged in the order of which topics are presented in the Ontario Revised Mathematics Curriculum of 2005. Secondary schools in the Niagara Catholic District School Board at present use the text Principles of Mathematics 9 by McGraw-Hill Ryerson.



The text has a website with student supports and additional resources that the student may find helpful. It can be accessed through the following web address:

http://highered.mcgraw-hill.com/sites/0070973199/student_view0/index.html

As well, an e-version of the text is available to all students registered in a Grade 9 Academic Mathematics class and their teachers. The text is available through the website:

http://www.mytextbook.ca/

Students require a passcode that can be obtained through their teacher.

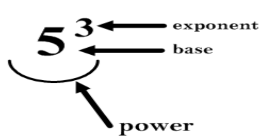
This guide was created in this fashion in the event a class uses a different primary resource, such as a textbook from another publisher, or a textbook alternative such as TIPS4RM

Number Sense and Algebra

Key Words:	
Algebraic Expression: a mathematical phrase made up of numbers and variables, connected by addition or subtraction signs (eg. $3x + 4$) Also called a **Polynomial**.	**Coefficient:** the number part of a term, also called the **numerical coefficient**
Constant Term: a term with only a numerical coefficient and no variable	**Degree of a Term:** the sum of the exponents on the variables in a term, also described as the number of variable factors in a term
Degree of a Polynomial: the degree of the highest degree term	**Distributive Property:** rule by which polynomials are multiplied, or expanded
Equation: a mathematical statement that shows two expressions are equal	**Formula:** describes an algebraic relationship that exists between two or more variables
Like Terms: terms that have identical variables	**Root:** the value for the variable that make the equation true, also called a **solution**
Scientific Notation: a convenient way of expressing very large or very small numbers, using a product of a number between 1 and 10 and a power of 10	**Solution:** the value for the variable that make the equation true
Term: an expression formed by the product of a number and a variable.	**Variable:** a quantity whose value can change, usually represented by a letter, also called a **literal coefficient**

Operating with Exponents

- **Substitution into and evaluation of expressions involving exponents**



Making sure students understand the difference between expressions $(-3)^4$ **and** -3^4

$(-3)^4 = (-3)(-3)(-3)(-3)$ $\qquad$ $-3^4 = -(3)(3)(3)(3)$

With rational bases, the exponent applies to both the numerator and the denominator

$$\left(\frac{2}{3}\right)^2 = \frac{(2)^2}{(3)^2} = \frac{4}{9}$$

- **The Exponent Rules**

$x^a x^b = x^{a-b}$ **Power Rule for multiplication of powers with the same base**

$\frac{x^a}{x^b} = x^a x^{-b} = x^{a-b}$ **Power Rule for division of powers with the same base**

$(x^a)^b = x^{ab}$ **Power Rule of a power rule**

These rules are used to simplify expressions in one or two variables with positive exponents.

Manipulating Expressions and Solving Equations

- **Simplifying and Evaluating numerical expressions involving integers and rationals**

RECALL: B.E.D.M.A.S.

Sample Questions:
Evaluate

a) $\frac{3}{8} - \left(-\frac{1}{4}\right)$ **b)** $\left(-3\frac{2}{9}\right) \div 1\frac{2}{3}$ **c)** $35 \div (-7) + (-3)(-5)$

- **Polynomials**

We name polynomials based on the number of terms.

		Polynomial Name
$4x$	One Term	Monomial
$3x+2$	Two Terms	Binomial
$9x^2-2x+1$	Three Terms	Trinomial

Polynomials are simplified by collecting like terms. Students are expected to work with polynomial terms that can involve up to two variables.

Examples: **a)** $6a - 2b + 3b + 2a$ **b)** $4x + 3xy + y + 5x - 2xy - 3y$

Expand polynomial expressions by multiplying a monomial by a polynomial, involving the same variable, using the distributive property.

Examples: **a)** $3p(p + 4)$ **b)** $5c(c^2 - 6c - 1)$

Expand and simplify polynomial expressions involving one variable.

Example: $-5(m^2 + 5m - 4) + 2m(m - 7)$

- **Solve first degree equations**

Students will be introduced to different methods for solving equations

Solve by Inspection – for one step equations where the solution is evident by "just looking at it".

Examples: **a)** $a + 4 = 7$ **b)** $5y = 35$ **c)** $\frac{k}{4} = 3$

Solve using the BALANCE Method – the balance method emphasizes what you do to one side of the equation, you must also do to the other side

Illustration: **In this method, we want to have students show the steps.**

Solve:

$2x + 3 = 11$

$2x + 3 - 3 = 11 - 3$ **Subtract three from both sides.**

$2x = 8$

$\frac{2x}{2} = \frac{8}{2}$ **Divide both sides by two.**

$x = 4$

Solve using OPPOSITE Operations – this method focuses on identifying an operation, and doing the opposite to the opposite side as the path to isolation of the variable.

Solve:

$2x + 3 = 11$

$2x = 11 - 3$ **Add 3 on the left becomes Subtract 3 on the right**

$2x = 8$

$x = \frac{8}{2}$ **Multiply by 2 on left becomes Divide by 2 on right**

$x = 4$

The goal is to have students move to a written solution like the second one above, but to have a firm understanding of the balance method and to make the connection between the two approaches. This is to avoid the idea that terms "move" from one side of the equation to the other.

- **Rearranging Formulas**

Example: **Given the equation** $y = mx + b$ **, rearrange to express** x **in terms of m,** y**, and b.**

- **Modelling with Algebra**

Express the following as an algebraic expression

a) three less than double a number
$2n - 3$

b) the square of a number increased by five
$x^2 + 5$

Solve the following problem:

Together, Blackie and Jessie have a mass of 72 kg. Blackie's mass is 4 kg less than Jessie's mass. What is each dog's mass?

Solution:

Let Jessie's mass be represented by the variable *m*.
So, Blackie's mass must be *m* – *4*.

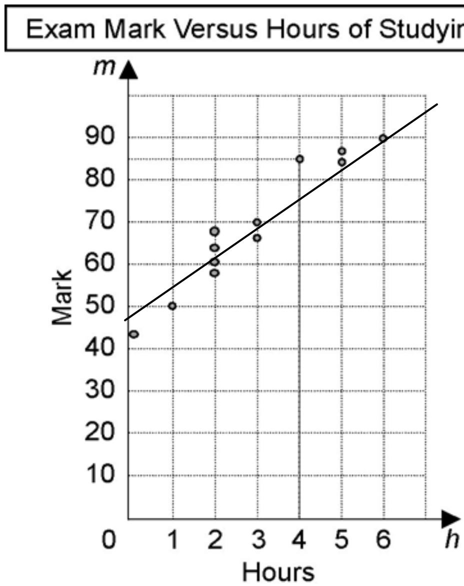
$m + m - 4 = 72$
$2m = 72 + 4$
$2m = 76$
$m = 38$

$\therefore$ **Jessie's mass is 38 kg and Blackie's mass is 34 kg.**

Linear Relations

Key Words:	
Bias: errors resulting from choosing a sample that does not represent the whole population	**Census:** a survey that includes each member of the population
Dependent Variable: a variable that is affected by some other variable	**Direct Variation:** a relationship between two variables in which one variable is a constant multiple of the other. This multiple is called the **constant of variation.**
Extrapolate: an estimated value that lies beyond the range of measured values in a data set	**Hypothesis:** a theory or statement that is either true or false
Independent Variable: a variable that affects the value of another variable	**Interpolate:** an estimated value that lies between two measured values in a set of data
Linear Relation: a relation between two variables that forms a straight line when graphed	**Line of Best Fit:** the straight line that comes closest to the points on a scatter plot. The line must show the trend in the data, and have roughly an equal number of points above and below the line.
Partial Variation: a relationship between two variables in which the dependent variable is the sum of a constant number and a constant multiple of the independent variable	**Population:** the whole group of people or items that are being studied
Primary Data: original data that a researcher gathers themselves for a particular experiment	**Random Sample:** a sample in which all members of the population have an equal chance of being chosen
Sample: any group of people or items that are selected from a population	**Sampling Methods:** these include **simple random sampling, systematic random sampling and stratified random sampling**
Secondary Data: data that someone else has already gathered for some other purpose	**Slope:** the measure of steepness of a line, calculated as the ratio $\frac{\text{rise}}{\text{Run}}$ In a linear relation, this value is also called the **constant of variation.**

Data Management for investigating relationships



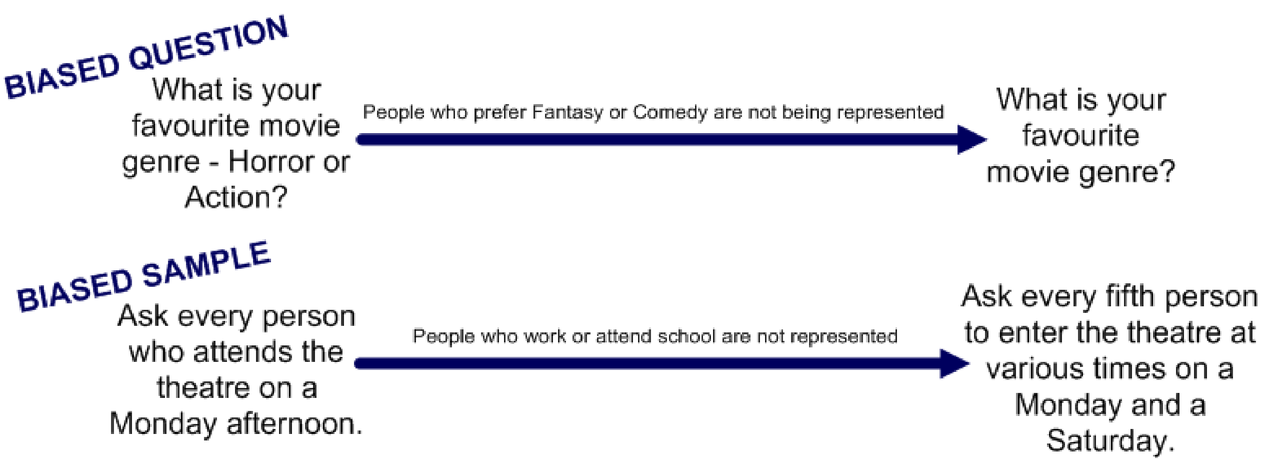
In the **scatter plot** shown, a relationship between Exam Mark and Hours of Studying is being explored. A **hypothesis** may have been, those that study more will get better grades.

Hours is the **independent variable,** and is shown on the horizontal axis, while Mark is the **dependent variable** and is shown on the vertical axis.

A **line of best fit** is drawn to show the relational trend in the variables.

A guess of a student's mark that studied for 3.5 hours would be an example of **interpolation,** while a guess at what a student would get if they studied for 7 hours would be an **extrapolation.**

- **A biased survey question or sample produces results that do not represent the population**



Randomly selected samples help to avoid the problems created by a biased sample. Random sampling techniques include:

Simple Random Sample: Choose a specific number of members randomly from a population. This could be done by selecting names from a hat, or assigning each member a number and conducting a lottery type draw.

Systematic Random Sample: Choose members from a population by first randomly selecting one member and then continue to select from the population at a fixed interval, such as every fifth person.

Stratified Random Sample: Divide the population into groups, or strata, then randomly select the same fraction of members from each group

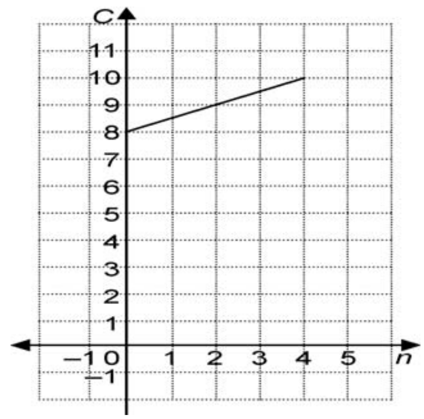
Characteristics of Linear Relations

- **Constructing tables of values, graphs and equations to represent a linear relation**

A large cheese pizza costs $8.00, plus $0.50 for each topping.

a) Make a table to represent the relation.
b) Graph the relation.
c) Write an equation to represent the relation.

Number of Toppings, *n*	Cost ($), *C*
0	8.00
1	8.50
2	9.00
3	9.50
4	10.00



$C = 0.5n + 8$

The above is an example of a Partial Variation. The graph of a partial variation does not pass through the origin, while a Direct Variation has its graph pass through the origin.

The graph shows that indeed the relation between a cost of pizza and the number of toppings is linear, this can also be observed through an examination of first differences.

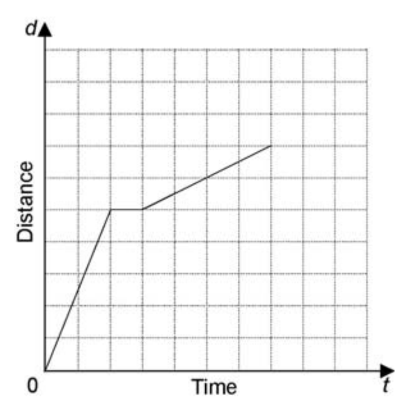
Number of Toppings, *n*	Cost ($), *C*	First Difference
0	8.00	8.5 – 8.0 = **0.5**
1	8.50	9.0 – 8.5 = **0.5**
2	9.00	9.5 – 9.0 = **0.5**
3	9.50	10.0 – 9.5 = **0.5**
4	10.00	

When the first differences are constant, this is property of a linear relation.

As well, if the independent variable increases by one, the first difference value will correspond to what is called the **constant of variation,** which is also the **slope** of the line.

- **Distance time graphs**

Michelle is late for school. She runs halfway to school,
then gets tired and stops for a short rest.
Then, she walks the rest of the way.
The graph to the right is a possible representation of this movement.



Students are to understand the meaning of direction, and rate of change of position from such a graph.

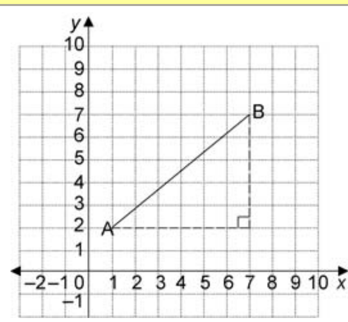
Analytic Geometry

Key Words:	
Intercepts: x- intercept, point at which a line crosses the x- axis and y – intercept is the place at which a line crosses the y axis	**Linear System:** a set of two or more linear equations considered simultaneously
Negative Reciprocals: the slopes of perpendicular lines are negative reciprocals to each other, the product of which is negative one	**Parallel Lines:** lines that run in the same direction and never cross
Perpendicular Lines: lines that intersect at right angles	**Point of Intersection:** the solution to a linear system, the point at which the two lines cross
Rate of Change: also called the slope of the line, the ratio of rise to run between two points	**Rise:** the vertical difference between two points
Run: the horizontal difference between two points	**Standard Form of a Linear Equation:** **Ax + By + C = 0**

Slopes of Lines

- $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{rise}}{\text{run}}$

  $m = \dfrac{5}{6}$



rise

run

Equations of Lines

- **Slope y- intercept Form** **y = mx + b**

Students should know how to Graph a line given these two

	Equation	Slope	*y*-Intercept
a)	$y = 4x + 1$	4	1
b)	$y = \frac{x}{2} - 3$	$\frac{1}{2}$	- 3

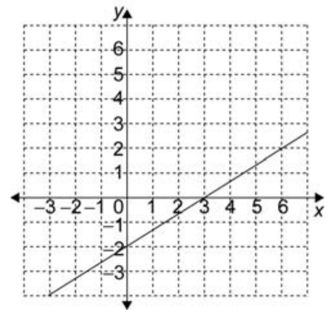
- **Standard Form of the equation of a line** **Ax + By + C = 0**

  2x + 3y – 5 = 0

  Students should know how to convert between forms.

- **Intercepts**

Identify the *x*- and *y*-intercept of the line.



x intercept is 3

y intercept is -2

By making connection that for x intercept, the y coordinate is 0 and that for the y intercept, the x coordinate is equal to zero, students should be able to identify the intercepts of a line given equation in either forms.

- **Parallel and Perpendicular Lines**

Parallel lines have equal slope $y = \frac{1}{3}x + 2$ $y = \frac{1}{3}x - 1$ are parallel lines

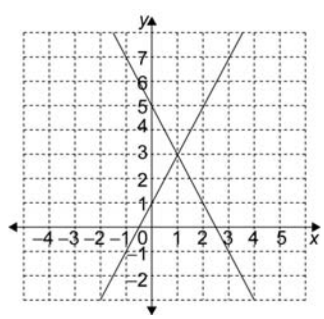
Perpendicular lines have slopes that are negative reciprocals

$2x - 3y + 12 = 0$ $3x + 2y = -4$ **are perpendicular lines**

$\mathbf{m_1 = \frac{2}{3}}$ $\mathbf{m_2 = \frac{-3}{2}}$ **Note: $m_1 \times m_2 = -1$**

Using Properties of Lines to Solve Problems

Solving a Linear System



Solution is (1,3)

Solving for a particular line

Find an equation for a line
a) with slope 6 passing through (–1, 4)
b) that passes through (–5, 0) and (5, 6)

Solving a problem

A retail store offers two different hourly compensation plans:
Plan A: $9.00 per hour
Plan B: $7.50 per hour worked plus a $4.50 shift bonus.

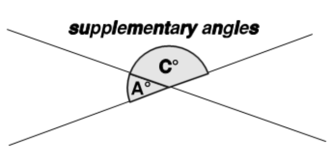
a) Graph the linear system. When would the earnings from the two plans be the same?
b) Describe a situation under which you would choose each plan.

Measurement and Geometry

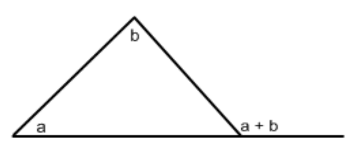
Key Words:	
Exterior Angle: angle formed on the outside of a geometric shape by extending one of the sides past a vertex	**Interior Angle:** angle formed on the inside of a polygon by two sides meeting at a vertex
Median: the line segment joining the vertex of a triangle to the midpoint of the opposite side	**Midpoint:** a point that divides a line segment into two equal segments
Optimization: process for finding a best, or optimal value or solution. This optimal value may be a minimum or a maximum.	**Polygon:** a closed figure made up of line segments
Prism: a polyhedron with two congruent parallel bases that are connected by lateral faces that are rectangles	**Pyramid:** a polyhedron whose base is a polygon, and whose sides are triangles that meet at a common vertex
Pythagorean Theorem: in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides Illustrated by the relation $c^2 = a^2 + b^2$	**Supplementary:** adding to 180 degrees
Surface Area: the number of square units needed to cover the surface of a three dimensional object	**Transversal:** a line intersecting two or more lines
Vertex: point where two or more sides meet	**Volume:** the amount of space that an object occupies, measured in cubic units

Investigating and Applying Geometric Relationships

- **Polygon Angle Properties**

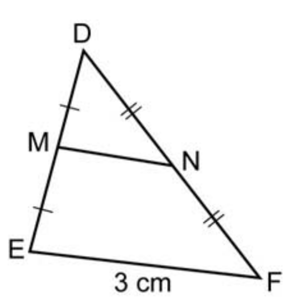


A + C = 180 °

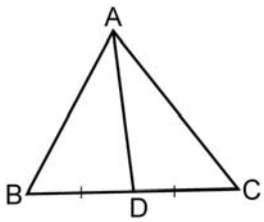


Exterior angles of a triangle

The sum of the interior angles of a triangle is 180 degrees.



Side MN = ½ EF



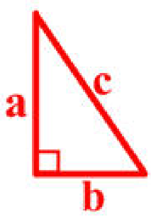
The median of a triangle bisects the area of the triangle

The sum of the interior angles of a polygon having n sides is equal to (n-2) × 180 degrees.

The sum of the exterior angles of a polygon is 360 degrees.

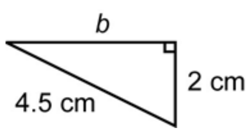
A regular polygon has all interior angles equal, and the lengths of each side equal.

Pythagorean Theorem



$$a^2 + b^2 = c^2$$

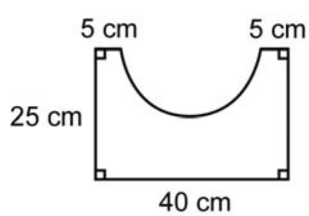
Students will solve problems with the above theorem.



Find the length of the unknown side.

Solving Problems involving Perimeter, Area, Surface Area and Volume

- **Composite Figures**



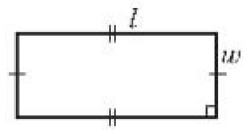
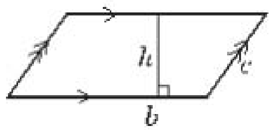
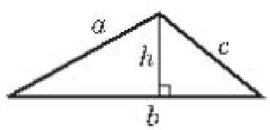
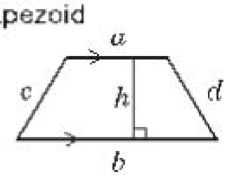
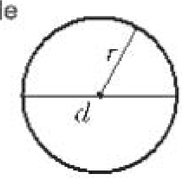
Find the perimeter and area of the above figure.

- **Two Dimensional and Three Dimensional Geometric Shapes**

The following two pages contain EQAO formula sheets containing all required formulas.

Formula Sheet

Grade 9 Academic

Geometric Figure	Perimeter	Area
Rectangle 	$P = l + l + w + w$ or $P = 2(l + w)$	$A = lw$
Parallelogram 	$P = b + b + c + c$ or $P = 2(b + c)$	$A = bh$
Triangle 	$P = a + b + c$	$A = \frac{bh}{2}$ or $A = \frac{1}{2}bh$
Trapezoid 	$P = a + b + c + d$	$A = \frac{(a + b)h}{2}$ or $A = \frac{1}{2}(a + b)h$
Circle 	$C = \pi d$ or $C = 2\pi r$	$A = \pi r^2$

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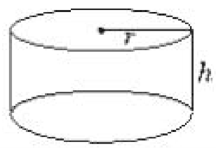
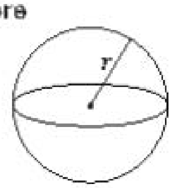
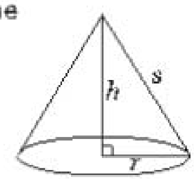
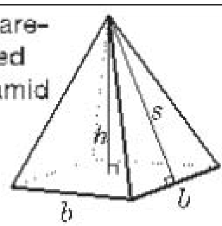
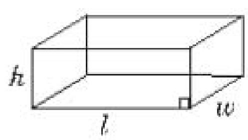
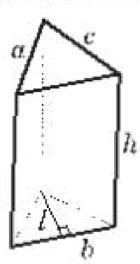
Geometric Figure	Surface Area	Volume
Cylinder 	$A_{base} = \pi r^2$ $A_{lateral\ surface} = 2\pi rh$ $A_{total} = A_{2\ bases} + A_{lateral\ surface}$ $= 2\pi r^2 + 2\pi rh$	$V = (A_{base})(\text{height})$ $V = \pi r^2 h$
Sphere 	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$ or $V = \frac{4\pi r^3}{3}$
Cone 	$A_{lateral\ surface} = \pi rs$ $A_{base} = \pi r^2$ $A_{total} = A_{lateral\ surface} + A_{base}$ $= \pi rs + \pi r^2$	$V = \frac{(A_{base})(\text{height})}{3}$ $V = \frac{1}{3}\pi r^2 h$ or $V = \frac{\pi r^2 h}{3}$
Square-based pyramid 	$A_{triangle} = \frac{1}{2}bs$ $A_{base} = b^2$ $A_{total} = A_{4\ triangles} + A_{base}$ $= 2bs + b^2$	$V = \frac{(A_{base})(\text{height})}{3}$ $V = \frac{1}{3}b^2 h$ or $V = \frac{b^2 h}{3}$
Rectangular prism 	$A = 2(wh + lw + lh)$	$V = (\text{area of base})(\text{height})$ $V = lwh$
Triangular prism 	$A_{base} = \frac{1}{2}bl$ $A_{rectangles} = ah + bh + ch$ $A_{total} = A_{rectangles} + A_{2\ bases}$ $= ah + bh + ch + bl$	$V = (A_{base})(\text{height})$ $V = \frac{1}{2}blh$ or $V = \frac{blh}{2}$

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Investigating Optimal Values of Measurements

- **Optimizing Rectangles**

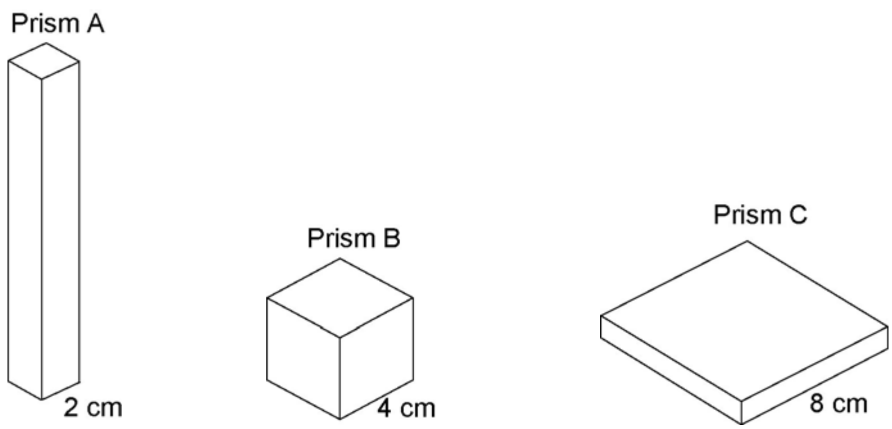
Sample Problem: A rectangle has perimeter 16 units.

a) List all the possible whole-number dimensions of the rectangle.
b) Which dimensions produce the rectangle with maximum area?
c) Describe the shape of the rectangle with maximum area.

- **Optimizing Prisms**

Sample Problem:

Each of these square-based prisms has volume 64 cm^3.



a) Find the height of each prism.
b) Find the surface area of each prism.
c) Order the prisms from least to greatest surface area.