MFM 2P - Grade Ten Applied Mathematics

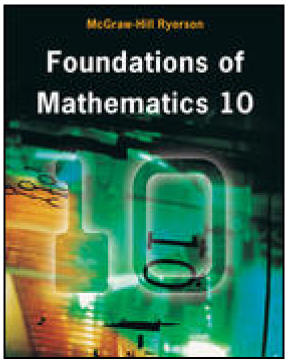
This guide has been organized in alignment with the 2005 Ontario Mathematics Curriculum. Each of the specific curriculum expectations are cross-referenced to the text book sections.

<u>Contents</u>

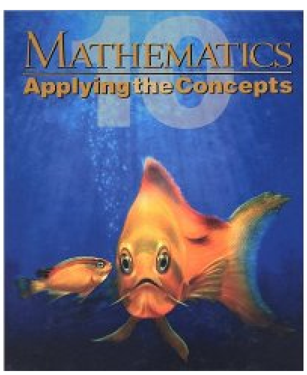
	Overall Expectations	Textbook Sections (Foundations of Mathematics 10 McGraw-Hill Ryerson)
Measurement and Trigonometry	**solve problems involving similar triangles**	**1.3, 1.4**
	solving problems involving the trigonometry of right triangles	**2.1, 2.2, 2.3, 2.4, 2.5**
	solving problems involving surface area and volume, using both metric an imperial systems of measurement	**1.1, 1.2, 9.1, 9.2, 9.3, 9.4, 9.5**
Modelling Linear Relations	**manipulating and solving algebraic equations**	**4.1, 4.2, 4.3, 4.4**
	graphing and writing equations of lines	**3.1, 3.2, 3.3, 3.4, 3.5**
	solving and interpreting systems of linear equations	**5.1, 5.2, 5.3, 5.4**
Quadratic Relations of the form $y = ax^2 + bx + c$	**manipulating quadratic expressions**	**7.1, 7.2, 7.3, 7.4**
	identifying characteristics of quadratic relations	**6.1, 6.2, 6.3, 6.4,**
	solve problems by interpreting graphs of quadratic relations	**8.1, 8.2, 8.3, 8.4**

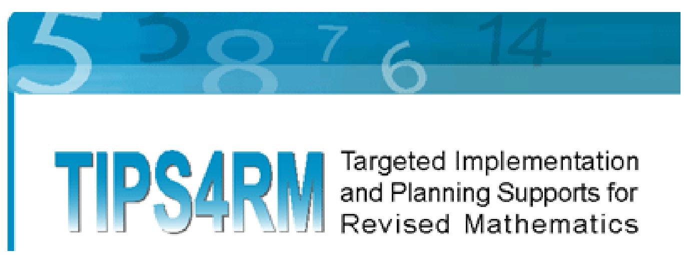
Introduction

This guide has been arranged in the order of which topics are presented in the Ontario Revised Mathematics Curriculum of 2005. Secondary schools in the Niagara Catholic District School Board as of September 2008 have been using the text Foundations of Mathematics 10, by McGraw-Hill Ryerson.



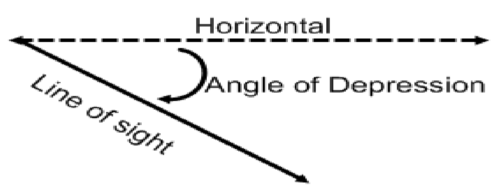
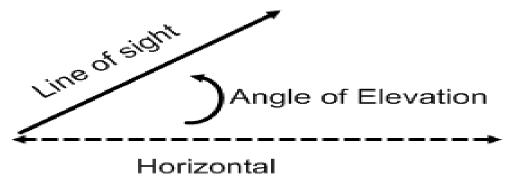
This guide was created in this fashion in the event a class uses a different primary resource, such as a textbook from another publisher, or a textbook alternative such as TIPS4RM. This may very well be the case, as this book is relatively new to the schools.. Many classrooms use the TIPS4RM as main resource, or may be using other teacher created resources.





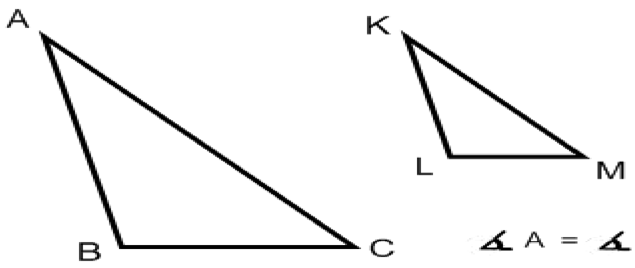
Measurement and Trigonometry

Key Words:

Angle of Depression: angle formed by the line of site and the horizontal when observing something that is below the horizontal	**Angle of Elevation:** angle formed by line of site and the horizontal when observing something that is above the horizontal
	
Cone: three dimensional object with a circular base and a curved side surface that tapers to a point	**Cylinder:** three dimensional object with two parallel circular bases
Corresponding Angles: a term with only a numerical coefficient and no variable	**Corresponding Sides:** rule by which polynomials are multiplied, or expanded
Imperial System: a system of measurement base on British units, such as feet, inches, pounds , etc.	**Metric System:** a system of measurement in which all units are based on multiples of 10
Prism: a solid with base and top faces that are congruent, parallel polygons and all other faces are parallelograms	**Pyramid:** a solid with a polygon base and all other faces are triangles
Pythagorean Theorem: in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides Illustrated by the relation $c^2 = a^2 + b^2$	**Proportional:** two quantities are proportional if they have the same constant ratio
Ratio: a comparison between two quantities	**Similar Triangles:** triangles with the same shape, but not necessarily the same size
Sphere: three dimensional ball shaped object where every point on the exterior is equidistant to the centre	**Surface Area:** total area of the external faces of a three dimensional object, measured in square units
Trigonometry: a word meaning triangle measurement, used to calculate lengths of sides and measures of angles in triangles	**Trigonometric Ratios:** the three primary trig ratios are sine, cosine and tangent
Volume: the amount of space occupied by a three dimensional object, measured in cubic units	

Similar Triangles

- Similar triangles have all corresponding angles equal and their corresponding side lengths are proportional, the two triangles shown are similar



$\angle A = \angle K$

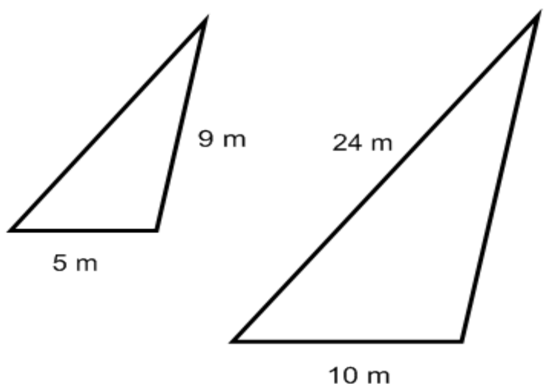
$\angle B = \angle L$

$\angle C = \angle M$

$$\frac{AB}{KL} = \frac{BC}{LM} = \frac{AC}{KM}$$

- **Determine the length of sides using proportional reasoning**

Sample Problem: The two triangles below are similar. Determine the lengths of the unknown sides.



Solution: Set up and solve equivalent ratios.

$$\frac{5}{10} = \frac{9}{x} = \frac{y}{24}$$

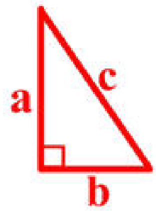
Several strategies can be shown as a means of solving for the unknown, including equivalent ratios, algebraic reasoning.

- **Solve problems involving similar triangles**

Sample problem: A girl needs to measure the height of a flag pole. She is 150 m tall. She finds that if she stands 12 m from the base of the flagpole, the top of her head touches the guy wire holding the flagpole up. The guy wire is anchored 15 m from the base of the pole. Draw and label a diagram, then determine the height of the pole.

Solving Problems involving the Trigonometry of Right Triangles

- **Pythagorean Theorem**

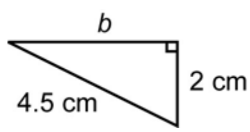


$$a^2 + b^2 = c^2$$

Often it may be a good approach to have students think of this relation as:

(hypotenuse)^2 = … then they know that identifying which side is the hypotenuse is the first step in solving for an unknown

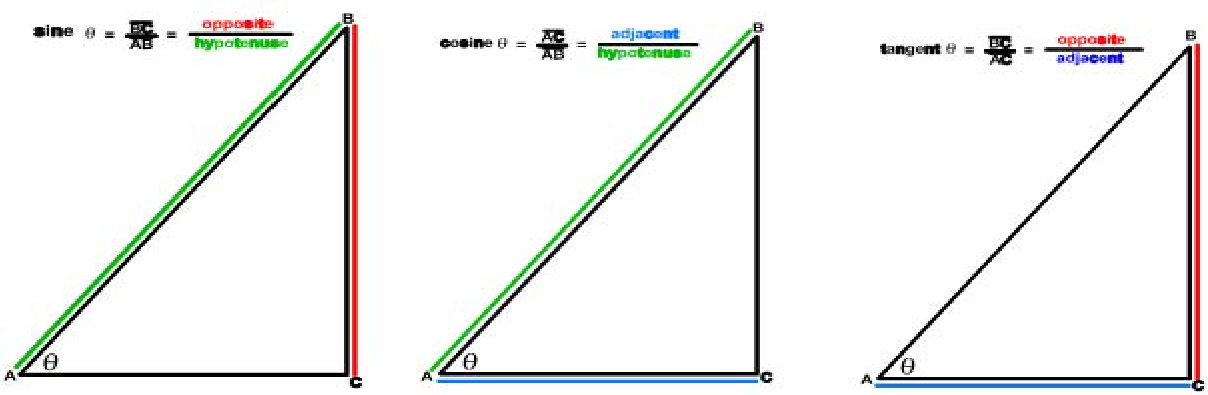
Students will solve problems with the above theorem.



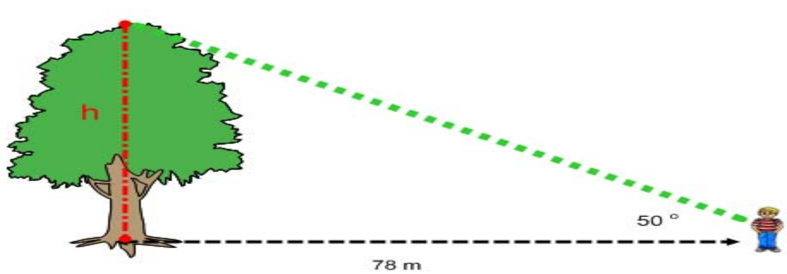
Find the length of the unknown side.

So here the solution would begin $(4.5)^2 = b^2 + 2^2$

- **Primary Trig Ratios**



Sample Problem: The angle of elevation of the top of a tree is 50°, for an observer that is standing 78 metres from the base of the tree. Determine the height of the tree.



$\tan 50° = \frac{h}{78}$

$h = 78 \times \tan 50°$

$h \approx 94$ m

Solving Problems involving Surface Area, Volume involving both the Metric and Imperial Systems

- **By estimations and conversions students work between the two systems of measurement for common, everyday measurements**

Equivalent Measures of Length	
1 meter (m)	39.37 inches (in.)
1 centimeter (cm)	0.39 in.
1 millimeter (mm)	0.039 in.
1 yard (yd)	91.44 centimeters (cm)
1 foot (ft)	30.48 cm
1 inch (in.)	2.54 cm

Household Measures (Approximate)	
1 drop	1/20 mL
1 teaspoon	5 mL
1 tablespoon	15 mL
1 cup	250 mL

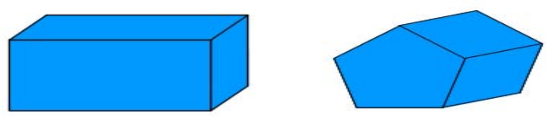
Weight and Apothecaries' Equivalents	
1 milligram (mg)	1/65 grain (1/60)
1 gram (g)	15.43 grains (15)
1 kilogram (kg)	2.20 pounds (avoirdupois)
1 pound (avoirdupois)	453.6 grams
1 grain (gr)	0.065 gram (60 mg)
1 ounce (1/16 pound)	28.4 grams

Fluid Equivalents	
1 fluid ounce (oz.)	29.57 mL (30)
1 pint (pt.) 16 fl. oz.)	473.2 mL (500)
1 pint, In the Imperial system	20 fluid ounces
1 quart (qt.)	946.4 mL (1000)
1 quart, In the Imperial system	40 fluid ounces
1 gallon (gal.)	3785.6 mL (4000)
1 gallon, In the Imperial system	160 fluid ounces

Examples: Estimate each of the following in the indicated units.

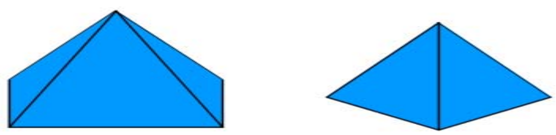
a)	4 miles	kilometres
b)	15 gallons	litres
c)	50 cm	inches

- **Volume and Surface Area problems**



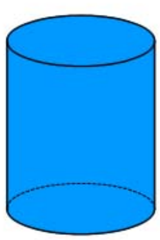
Volume for Prisms:

Area of Base x Height



Volume for Pyramids:

$\frac{(\text{Area of Base x Height})}{3}$

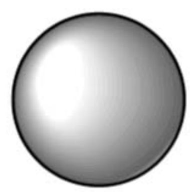


Volume for Cylinder:

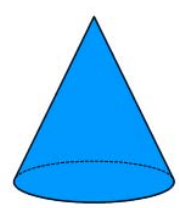
Area of Base x Height = $\pi r^2 h$

Surface Area of Cylinder:

$2\pi r^2 + 2\pi rh$

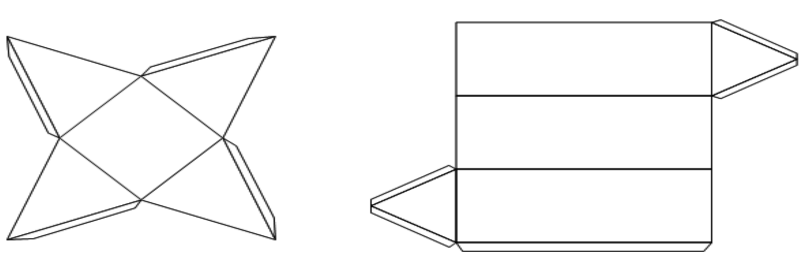


Volume of a Sphere: $\frac{4\pi r^3}{3}$



Volume of a Cone: $\frac{\pi r^2 h}{3}$

The above included formulas are those that the students are responsible for in solving problems in this 3-D geometry unit. Focus is not so much on memorization of the formulas, but rather their application. Net diagrams, like the one below may be used to help develop some of the surface area formulas.



Modelling Linear Relations

Key Words:	
Coefficient: a number that is multiplied by a variable; in the term 3x, 3 is the coefficient	**Constant Term:** a numerical term which cannot change; it does not contain a variable
Elimination Method: an algebraic method for solving a system equations in which the equations are added or subtracted to eliminate one variable Another algebraic method is called **Substitution Method**, in which one equation is substituted into the other	**Formula:** describes an algebraic relationship between two or more variables
Linear Relation: a relation between two variables that forms a straight line when graphed, other wise the relation is a called a **Non-linear Relation.**	**Linear System:** a set of two or more linear equations that are considered at the same time
Point of Intersection: the point at which two lines cross; the coordinates of the point of intersection satisfies both equations; also called the solution to a system of equations	**Rate of Change:** also called the slope of the line, the ratio of rise to run between two points
Rise: the vertical difference between two points	**Run:** the horizontal difference between two points
Standard Form: a linear equation of the form Ax + By + C = 0	**Variable Term:** a term that includes a letter or symbol to represent an unknown value

Manipulating and Solving Algebraic Equations

- **Solve first degree equations including those with fractional coefficient**

Samples: Solve and Verify $\frac{x}{2}+4=3x-1$

Solve $3\,(x-1) = 5\,(x-2)$

Rearrange for t $A = P\,(1 + rt)$

* a focus in solving equations will be the showing of all steps

- **Convert the equation of a line from Standard for into y = mx + b form**

Rewrite $2x + 3y - 6 = 0$ into $y = mx + b$ form

* this is to take advantage of the fact that slope and y intercept are easily obtained

Writing Equations and Graphing Lines

- **Constructing tables of values, graphs and equations to represent a linear relation**

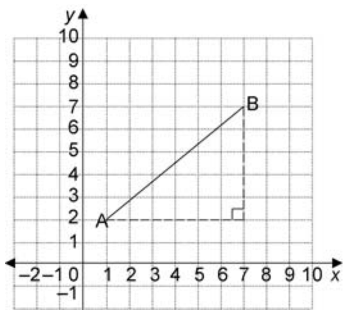
Solving a problem

A retail store offers two different hourly compensation plans:
Plan A: $10.00 per hour
Plan B: $7.00 per hour worked plus a $6.00 shift bonus.

a) Write equations to represent the two plans
b) Graph the linear system.
c) When would the earnings from the two plans be the same? How can you tell?
d) Describe a situation under which you would choose each plan.

- Rate of change $= \frac{\text{rise}}{\text{run}}$

$= \frac{5}{6}$



rise

run

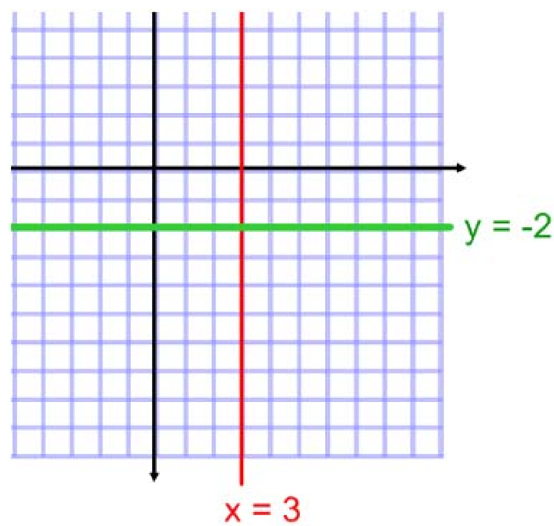
A constant rate of change is a straight line segment on the graph.

- **Special Equations**

Graph the special case linear equations:

x = 3

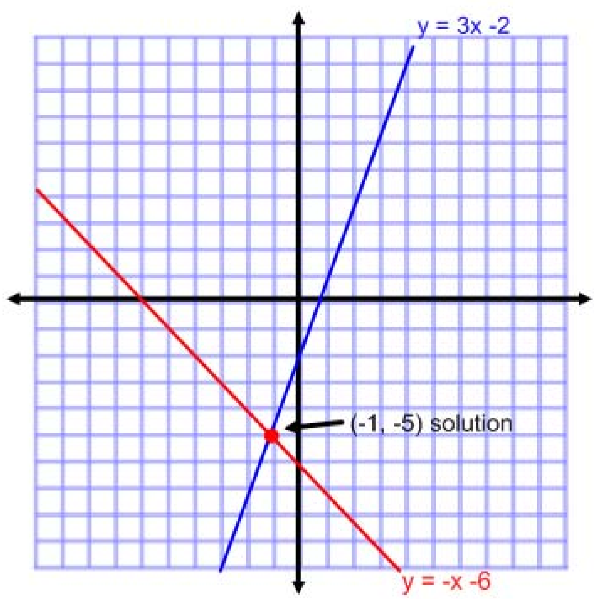
y = -2



Solving Systems of Equations

- A "system" of equations is a set or collection of equations that are dealt with simultaneously. Linear systems of equations can be solved both graphically and algebraically.
  - Graphically : Find the point of intersection of the two lines.

$$y = 3x - 2 \qquad y = -x - 6$$



  - Algebraically : The methods of substitution and elimination are learned.

In **substitution**, one equation is rearranged and substituted into the other to create an equation in one variable that can be solved for.

Example: $2x - 3y = -2$
$4x + y = 24$

In the equation $4x + y = 24$, rearrange to isolate for y and get $y = -4x + 24$

This is then substituted into the first equation, $2x - 3(-4x + 24) = -2$

This equation is now in one variable and can be solved for x. ($x = 5$)

Now this value for x can be substituted into either of the two original equations to find y.

In **elimination**, equations are adjusted so that by adding or subtracting the equations, one of the two variables is eliminated. This will create an equation in one variable that can be solved for.

Example: $2x - 3y = -2$
$4x + y = 24$

If the second equation is multiplied by 3, the result would be $12x + 3y = 72$

If this is now added to the first equation,

$$\begin{array}{r} 2x - 3y = -2 \\ \underline{12x + 3y = 72} \\ 14x \quad = 70 \end{array}$$

(the y terms are eliminated)

This equation is now in one variable and can be solved for x. ($x = 5$)

Now this value for x can be substituted into either of the two original equations to find y.

Sample Problem: Maria has been hired by Company A with an annual salary, S dollars, given by $S = 32\,500 + 500a$, where a represents the number of years she has been employed by this company. Ruth has been hired by Company B with an annual salary, S dollars, given by $S = 28\,000 + 1000a$, where a represents the number of years she has been employed by that company.
Describe what the solution of this system would represent in terms of Maria's salary and Ruth's salary.
After how many years will their salaries be the same?
What will their salaries be at that time?

Quadratic Relations

Key Words:

Algebraic Expression: a mathematical phrase made up of numbers and variables, connected by addition or subtraction signs (eg. $3x + 4$) Also called a **Polynomial**.	**Axis of Symmetry:** the line of symmetry that passes through the vertex of the parabola
Difference of Squares: a special quadratic of the form $a^2 - b^2$	**Distributive Property:** rule by which polynomials are multiplied, or **expanded**
Factoring: describes an algebraic process of converting a polynomial into a product of two polynomials. Opposite to expanding.	**First Differences:** the differences between the y-values that correspond to consecutive x-values **Second Differences** are the differences between consecutive first differences
Perfect Square Trinomial: trinomial of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$	**Parabola:** the graph of a quadratic relation
Quadratic expression: an expression of the form $ax^2 + bx + c$	**Quadratic relation:** a relation between two variables that appear as a parabola when graphed; relation of the form $y = ax^2 + bx + c$
Vertex: the turning point of the parabola	**Zeros:** the x intercept of a quadratic relation

Manipulating Quadratic Expressions

- **Polynomials**

  Expand and Simplify second degree polynomial expressions such as:

  $(3x + 4)(2x - 5)$ $\quad\quad$ $(x - 3)^2$

  Factor Polynomial expressions

  Several factoring techniques are covered:

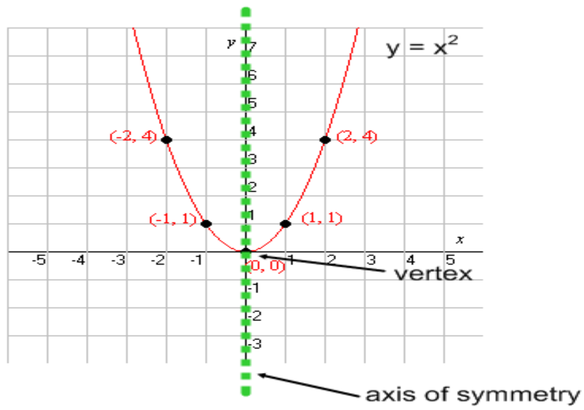
  Common Factoring (factoring a G.C.F.) $\quad$ Example: $10x^2 + 15x$ $= 5x(2x + 3)$ $\quad$ 5x is the greatest common factor between the two terms

  Factoring Simple Trinomials $x^2 + bx + c$ (Where the number in front of *x* squared is 1) $\quad$ Example: $x^2 + 9x + 20$ $= (x \underline{+ 4})(x \underline{+ 5})$ $\quad$ Look for a pair of integers so their sum is b & product is c

  Difference of Squares $\quad$ Example: $x^2 - 49$ $= (x + 7)(x - 7)$

Characteristics of Quadratic Relations

- Identify the key features of a parabola



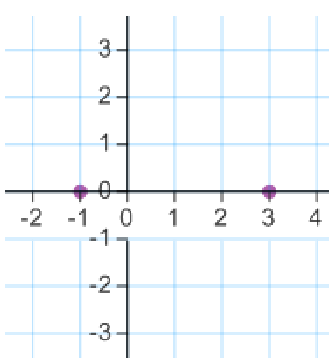
- **First and Second differences**

x	y	First Difference	Second Difference	
1	1			
2	4	4 - 1 = 3		
3	9	9 - 4 = 5	5 – 3 = 2	**In a quadratic relation, the second differences will all be equal**
4	16	16 - 9 = 7	7 – 5 = 2	
5	25	25 - 16 = 9	9 – 7 = 2	

- **Use factoring to make connections to the zeros/x intercepts of a quadratic in the form $y = (x-r)(x-s)$**

For Example: Find the x intercepts and sketch the quadratic $y = x^2 - 2x - 3$

$y = x^2 - 2x - 3 \quad = (x - 3)(x + 1)$



So the x intercepts of the graph will be x = 3 and x = -1

Axis of symmetry must lie exactly half way between these points, so vertex must occur at x = 1.

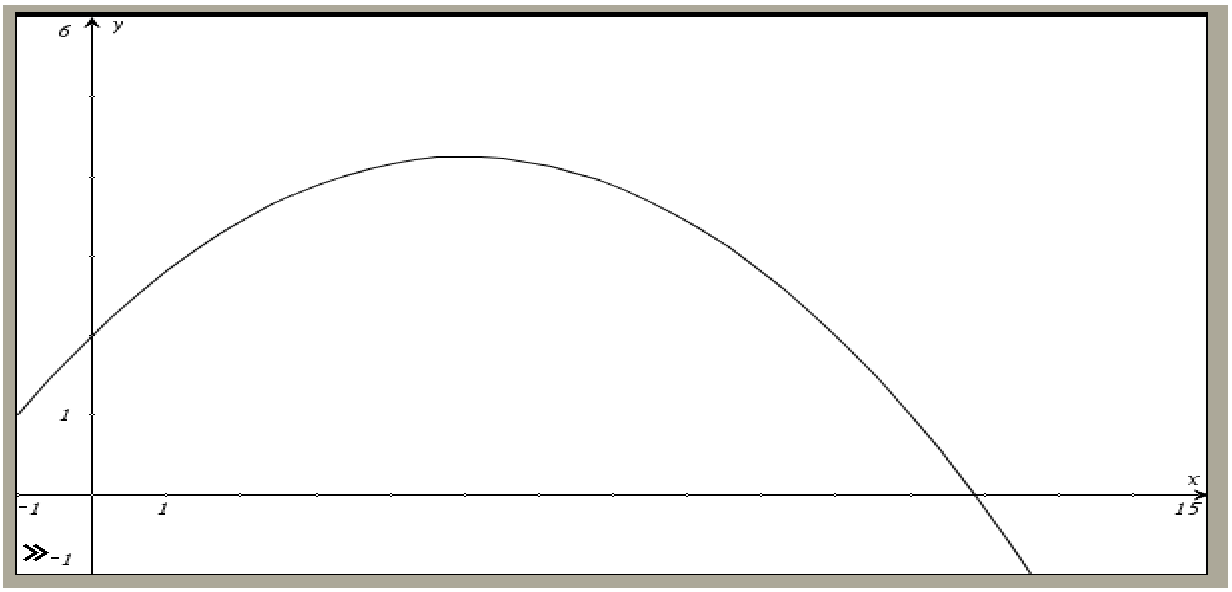
Sub x = 1 into the equation and get y = -4, so vertex has coordinates (1, -4)

Solving Problems by interpreting Graphs of Quadratics

Sample Problem:

The path of a basketball shot through the air can be modelled by the equation $h = -0.09d^2 + 0.9d + 2$, where h is height in metres and d is the horizontal distance of the ball from the player in metres.

The graph of the equation is given below:



a) Determine the maximum height of the ball.
b) Determine the horizontal distance of the ball from the player when it is at its maximum height.
c) Determine the height of the ball the moment it is released by the player.

Solutions: Problems such as the one above allows for several approaches, however an emphasis in this course is to take advantage of technology, such a graphing calculator or software program or a spreadsheet approach, creating a table of values